Using a large-angle gauge to select trees for measurement in variable plot sampling

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Abstract: Variable plot sampling has been widely used for many years. It was recognized, early in its application, that the process of getting stand volume could be divided into two components, counting trees to get basal area per unit area and measuring trees to get volume/basal area ratios (VBARs). It was further recognized that these two components had different amounts of variation and therefore should be sampled at different intensities. The fact that basal area per unit area is almost always more variable than the VBARs of individual trees has led to the widespread practice of counting trees on all plots and subsampling trees for VBAR measurements, typically by measuring all the trees on every third or fourth plot. This article presents an alternative, the "big BAF method," which uses a larger basal-area-factor angle gauge to do a second sweep of each plot to select the trees to be measured for VBAR. This procedure spreads the tree measurements throughout the stand and is thus more statistically efficient. The method is simple to apply, requires no additional computations, and is easy to audit. Two case-study examples are used to demonstrate the method.

Résumé : La place-échantillon à superficie variable a été largement utilisée depuis plusieurs années. Dès le début de son utilisation, il a été reconnu que le processus pour obtenir le volume sur pied pouvait être séparé en deux composantes: le décompte des arbres pour obtenir la surface terrière par unité de surface et la mesure des arbres pour obtenir le rapport du volume sur la surface terrière (RVST). Il a de plus été reconnu que ces deux composantes varient différemment et devraient donc être échantillonnées suivant différentes intensités. Le fait que la surface terrière par unité de surface soit presque toujours plus variable que le RVST des arbres individuels, a engendré la pratique courante qui consiste à compter les arbres dans toutes les places-échantillons et de sous-échantillonner les arbres pour la mesure du RVST, typiquement en mesurant tous les arbres à toutes les 3^e ou 4^e places-échantillons. Une méthode alternative, appelée méthode du grand facteur de prisme, utilise un plus grand facteur de prisme pour faire un second balayage de la place-échantillon et sélectionner les arbres à mesurer pour le RVST. En répartissant la mesure des arbres dans l'ensemble du peuplement, cette méthode est statistiquement plus efficace. Elle est simple d'application, n'exige aucun calcul additionnel et est facile à vérifier. Deux études de cas sont utilisées pour illustrer la méthode.

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Introduction

Variable plot sampling (also called point, horizontal point, angle-count, prism, or Bitterlich sampling) is widely used for forest inventory. Bitterlich (1948) developed the idea of using a horizontal angle gauge for estimating basal area per unit of land area by simply counting trees with diameters that subtend angles larger than the horizontal angle gauge. Grosenbaugh (1952, 1958) introduced this idea to North American foresters and extended it to provide estimates of volume and other stand variables from measured trees. The basic approach can be expressed in the following two formulas:

$$[1a]\quad \text{Basal area per unit area} = \text{average tree count} \times \text{BAF}$$

$$[1b]\quad \text{Volume per unit area} = \text{basal area per unit area} \times \text{average VBAR}$$

where BAF is the basal-area factor (m^2/ha) of the angle gauge used; and VBAR is the average ratio of tree volume/ tree basal area (also known as a mean-of-ratios estimator). Bitterlich (1984) provided a detailed description of the method and its development.

Bell and Alexander (1957) showed how the standard error in percent (SE%) of the estimated volume could be computed by combining the SEs for basal area (tree count) and VBARs and using the formula that is commonly known as "Bruce's method" (Goodman 1960):

$$[2]\quad \text{SE}_{\text{combined}}\% = \sqrt{\text{SE}_{\text{TC}}\%^2 + \text{SE}_{\text{VBAR}}\%^2}$$

This formula assumes that the tree counts and VBARs are statistically independent, and it is simplified by dropping a small, negative third term ($\text{SE}_{\text{TC}}\%^2 \times \text{SE}_{\text{VBAR}}\%^2$). The assumption of independence is commonly made in this application and has proven an adequate approximation for most cruise planning and reporting needs. In addition, the combined SE% of a typical cruise will tend to be conservatively large, as systematic sampling is almost always used for forest inventory sampling, but computations are done as if it were a random sample. Bruce (1961) pointed out that the variability for basal area is usually higher than for VBAR

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and demonstrated that one can make a greater reduction in the combined SE% by reducing that larger component. Bruce gave the following example to demonstrate this principle:

$$\mathrm{SE}_{\mathrm{combined}}\% = \sqrt{10\%^2 + 2\%^2} = 10.198\%$$

$$\mathrm{SE}_{\mathrm{combined}}\% = \sqrt{5\%^2 + 2\%^2} = 5.385\%$$

$$\mathrm{SE}_{\mathrm{combined}}\% = \sqrt{10\%^2 + 1\%^2} = 10.050\%$$

This clearly shows that reducing the larger SE% (as could be done through additional fieldwork) from 10% to 5% makes a greater impact on the overall combined SE% than reducing the smaller value from 2% to 1%. In the case of variable plot samples in which the larger SE% is almost always associated with basal area, this would suggest the desirability of taking many plots where just basal area is estimated from tree counts and measuring only a subsample of those trees to determine the average VBAR. Bruce (1961) pointed this out and suggested that one way to do this would be to measure the trees on every third or fourth plot where a tree count was made to estimate basal area. The idea of subsampling tree measurements was also discussed by Johnson (1961), Palley and Horwitz (1961), and Beers and Miller (1964). The procedure of measuring all the trees on a subsample of the count plots has been practised for about 40 years in the Pacific Northwest. It has also been referred to as "point double sampling" (Oderwald and Jones 1992).

This process of measuring all trees at every third or fourth point is a form of cluster sampling. The advantage of a cluster sample is that it may be more economical to take measurements at one location, rather then distributing them. A disadvantage of this approach is that when the individuals within the cluster are similar, more measurements in the cluster will not proportionately improve the precision of the average. This is a problem whether the cluster is selected with a fixed radius or a variable-radius plot is used. If the trees that are near each other have similar VBARs, it would be more statistically efficient to distribute the measurements through the stand.

A number of methods have been used to distribute the measured trees through the stand. Some are incorrect, and others, while technically correct, are difficult to apply in the field. Some suggested but incorrect methods are to measure the first "in" tree on a plot, the first "in" tree from the north, or the closest "in" tree to the plot center on each sample point. These procedures are all biased (Iles 2003). Selecting the closest tree tends to oversample smaller trees, which tend to have smaller VBARs, and this could result in an underestimate of stand volume. Selecting the first tree from the north (or any other direction) tends to bias the selection toward isolated trees. Measuring only one tree on a plot will weight the average VBAR toward plots with smaller tree counts.

One of many possible correct procedures would be to select trees for measurement with equal probability from all "in" trees. This could be done with random numbers or any similar probability scheme. Another way would be to systematically select every *i*th tree that gives the desired proportion of count plots to subsampled trees measured for VBAR. Unless a data recorder is used, these methods can cause some messy bookkeeping, particularly if the selection ratio is different between species.

In this article, we suggest an alternative way to subsample the trees to be measured for VBAR: using an additional large BAF angle gauge. The method is applied in two steps. First, the usual BAF angle gauge is used to get a tree count to estimate basal area at each sample point. Second, another angle gauge, usually with a much larger BAF, is used to select the trees to be subsampled and measured for VBAR. For example, if the desire is to select roughly every fifth tree to measure for VBAR, an angle gauge with a BAF five times larger than the one used to obtain tree counts would be used. Husch et al. (1982) suggested using a larger BAF to select a subsample of trees to increment core for growth measurement but did not extend this to measuring VBAR. The "big BAF method" we are suggesting is easy to apply, requires no computational changes, and is easily adjusted to optimally subsample trees for VBAR measurement. The first written description of this particular method of "distributed VBARs" seems to be Bell et al. (1983), with a more developed description provided by Iles (1989). In this article, we give two case-study examples of the use of the big BAF method and discuss the application and benefits of the method.

Selecting sample sizes

Selecting the sample size required for estimating basal area using variable plot sampling requires the use of an appropriate-sized BAF angle gauge. The BAF to use for tree counts can be computed by dividing the stand's expected basal area per unit area by the desired average number of "in" trees on each plot. A common target for the desired average number of "in" trees is 4–10 per plot. Experience has shown that plots with smaller tree counts have excessive variability and require more effort to get a desired SE, and plots with larger tree counts tend to be subject to increasing personal error from missing trees and edge effects (Iles 1989). If applied correctly, the choice of any BAF will give unbiased estimates of basal area, so it becomes a question of what can be applied most correctly and most efficiently (in that order).

The proportion ($P_{\mathrm{TC/VBAR}}$) of plots from which tree counts are obtained to the number of trees measured for VBAR can be computed by the following formula (Bell et al. 1983; Bell and Dilworth 2002):

$$[3] \qquad P_{\mathrm{TC/VBAR}} = \frac{\mathrm{CV}_{\mathrm{TC}}\sqrt{\$\mathrm{VBAR}}}{\mathrm{CB}_{\mathrm{VBAR}}\sqrt{\$\mathrm{TC}}}$$

where $\mathrm{CV}_{\mathrm{TC}}$ and $\mathrm{CV}_{\mathrm{VBAR}}$ are the coefficients of variation for tree count and VBAR; and \$TC and \$VBAR are the costs of measuring a plot for basal area (taking a tree count) and for measuring a tree's VBAR (e.g., measuring diameter, height, and possibly form and grade), respectively. The specific sample sizes for each part of the process are computed as

$$[4] \qquad n_{\mathrm{VBAR\,trees}} = \frac{\mathrm{CV}_{\mathrm{TC}}^2 + (\mathrm{CV}_{\mathrm{VBAR}}^2 \times P_{\mathrm{TC/VBAR}})}{\mathrm{SE}_{\mathrm{combined}}\%^2 \times P_{\mathrm{TC/VBAR}}}$$

for the number of trees to measure for VBAR and

$$[5] \qquad n_{\mathrm{TC\ per\ point}} = P_{\mathrm{TC/VBAR}} \times n_{\mathrm{VBAR}}$$

for the number of plots on which to obtain a tree count to estimate basal area. These equations were derived from the example of minimizing the total cost of a cluster sample found in most sampling texts (Scheaffer et al. 1990). If value, rather then volume, is the pertinent issue, then the CV of the dollars/basal area ratio ($\mathrm{CV}_{\$\mathrm{BAR}}$) can be used in this calculation, rather than $\mathrm{CV}_{\mathrm{VBAR}}$ (Iles and Bell 1986).

In the historical application, in which all trees are measured on a proportion of the plots, the number of plots to measure for VBAR would be determined by dividing the number of VBAR trees needed by the anticipated total tree count. If 100 VBAR measurements are needed and 600 trees are expected to be counted during the cruise, then one in six plots would be completely measured. These plots would then be scattered throughout the stand being sampled, usually in a systematic way. However, most people feel insecure about measurements taken at these large intervals. In reaction to that insecurity, they often measure many more plots, resulting in increased cost.

Methods

Data from two variable probability sampling workshops are used to illustrate the application and results of the big BAF method. These will be called the Tahoe and Oregon State University (OSU) data sets. In both of the workshops, each crew visited 20 sample points. At each sample point, crews first did a tree count using an angle gauge with a BAF that would give an average tree count of 4–10 trees for the stand conditions at each site. Next a larger BAF angle gauge was used at the same sample point to select “in” trees to be measured for diameter at breast height (DBH, in inches) and total height (THT, in feet), which were used to compute gross VBAR. To allow additional comparisons, crews also measured all trees that were “in” with the smaller BAF at their first sample point. The first sample points for each of the crews were then combined to create a data set in which all trees were measured on all of the sample points. This was considered the standard method, which would be compared with the methods in which trees were subsampled. The stand conditions for the two cruise areas are summarized in Table 1.

Table 1. Summary of the two sample stands.

Stand	Trees/ha	Basal area (m^2/ha)	Quadratic mean DBH (cm)	Gross volume (m^3/ha)
Tahoe	205.6	40.3	50.9	368
OSU	143.1	84.5	86.7	1466

Note: DBH, diameter at breast height; OSU, Oregon State University.

The Tahoe cruise was done in 1997, near South Lake Tahoe, in California. This was a Jeffery pine (*Pinus jeffreyi*) stand with less than 3% of the basal area in other species (lodgepole pine (*Pinus contorta*) and white fir (*Abies concolor*)). By means of a relascope, trees were counted at DBH with a 4.59 m^2/ha (20 ft^2/acre) BAF, and measurement trees were selected with a 20.66 m^2/ha (90 ft^2/acre) BAF prism. Tree DBH and THT were measured, then volume in cubic metres to a 15.24-cm (6-in.) top diameter inside bark was computed using the taper equation of Walters and Hann (1986) for ponderosa pine (*Pinus ponderosa*). The VBARs for the measured trees were computed by dividing the calculated volumes by the tree basal area at DBH.

The OSU cruise was done in 1999, on the Oregon State University College of Forestry Research Properties, near Corvallis, Oregon. All trees in the cruise were Douglas-fir (*Pseudotsuga menziesii*). By means of a relascope, trees were counted at DBH with a 9.18 m^2/ha (40 ft^2/acre) BAF, and measurement trees were selected with a 57.59 m^2/ha (250 ft^2/acre) BAF. Again, DBH and THT were measured for selected trees, and the volume in cubic metres was computed to a 15.24-cm (6-in.) top diameter inside bark with the taper equation of Walters and Hann (1986) for Douglas-fir. VBARs were computed as described for the Tahoe data set.

As in any learning activity, mistakes were made. For this demonstration, we assumed that all tree counts and tree measurements were correct. The only exception was plot 15 of the Tahoe data set. On this plot, a tree count was made with the smaller BAF, but only the trees that were “in” with the large BAF were measured (i.e., not all trees on the plot were measured). To make comparisons for all trees being measured in this data set, the CV for VBAR was computed with only the measured trees (four fewer than expected), but the costs were computed as if all the trees had been measured. This should make little difference in the calculated CV.

A rate of \$60.00/h was used to compare costs. It was assumed that the average travel time between plots was 4 min (\$4.00) per plot. The average time to get a tree count was 2 min (\$2.00) per plot, and the average time to measure a tree was 2 min (\$2.00) per tree. Optimal sample sizes were computed by using eqs. 3, 4, and 5 (above). In all cases, computed sample sizes (number of plots or trees) were rounded up to the nearest integer.

Results

The sampling results for Tahoe are given in Table 2, and those for the OSU example are given in Table 3. At Tahoe, the 4.59 m^2/ha BAF gave an average tree count of 9.1 trees per plot and 182 total trees. At OSU, the 9.18 m^2/ha BAF gave an average tree count of 9.2 trees per plot and 184 total trees. At both locations, the basal area (tree count) was more variable than VBAR. The stand at Tahoe was more variable than OSU for both measurements: at Tahoe, the $\mathrm{CV}_{\mathrm{TC}}$ was nearly 62%, whereas it was 33.0% at OSU; and the VBAR trees had a $\mathrm{CV}_{\mathrm{VBAR}}$ of about 23% at Tahoe and 16% at OSU.

As shown in Table 2 for the Tahoe data set, if all of the 182 trees selected with the 4.59 m^2/ha BAF angle gauge had been measured for VBAR, the resulting combined SE% would have been 13.9%, at a total cost of \$476 (the standard for making comparisons). Measuring only about half of the trees (all trees on either the even or odd numbered plots) only slightly increased the combined SE%, to 14.0%, but the fewer measured trees reduced the total cost to between 60% and 65% of the standard full-measured option. By using a 20.66 m^2/ha BAF to select the trees to measure, a total of 39 trees (21%) were measured, giving a slightly larger combined SE% of 14.4% on the same tree-count plots, but this reduced the cost to 42% of the standard. The same SE% of 14% could have been optimally obtained with the big BAF method by doing tree counts on about 28 plots and measur-

Table 2. Results from Tahoe workshop using a 4.59 m^2/ha BAF for selecting “in” trees and a 20.66 m^2/ha BAF for selecting VBAR trees.

Situations	No. of sample points	No. of counted “in” trees[a]	No. of VBAR trees	CV_{TC} (%)	CV_{VBAR} (%)	Combined SE% (%)	Total cost ($)
All trees	20	182	178	61.6	23.1	13.9	476
Optimum	28	245	11	61.6	23.1	13.9	190
Even plots	20	182	95	61.6	23.7	14.0	310
Optimum	27	244	11	61.6	23.7	14.0	184
Odd plots	20	182	83	61.6	22.5	14.0	286
Optimum	27	241	10	61.6	22.5	14.0	182
Big BAF	20	182	39	61.6	26.7	14.4	198
Optimum	27	239	12	61.6	26.7	14.4	186

Note: The calculated number of samples is rounded up to the nearest whole unit. BAF, basal-area factor; CV_{TC}, coefficient of variation for tree count; CV_{VBAR}, coefficient of variation for VBAR; VBAR, volume/basal area ratio.

[a] Optimum number of counted “in” trees is based on the average tree count of 9.1 trees per plot.

Table 3. Results from OSU workshop using a 9.18 m^2/ha BAF for selecting “in” trees and a 57.59 m^2/ha BAF for selecting VBAR trees.

Situation	No. of sample points	No. of counted “in” trees[a]	No. of VBAR trees	CV_{TC} (%)	CV_{VBAR} (%)	Combined SE% (%)	Total cost ($)
All trees	20	184	184	33.0	16.1	7.5	488
Optimum	29	266	15	33.0	16.1	7.5	204
Even plot	20	184	84	33.0	16.0	7.6	288
Optimum	28	258	14	33.0	16.0	7.6	195
Odd plots	20	184	100	33.0	16.6	7.6	320
Optimum	29	260	14	33.0	16.6	7.6	202
Big BAF	20	184	39	33.0	14.7	7.8	198
Optimum	27	239	12	33.0	14.7	7.8	180

Note: The calculated number of samples is rounded up to the nearest whole unit. BAF, basal-area factor; CV_{TC}, coefficient of variation for tree count; CV_{VBAR}, coefficient of variation for VBAR; OSU, Oregon State University; VBAR, volume/basal area ratio.

[a] Optimum number of counted “in” trees is based on the average tree count of 9.2 trees per plot.

ing about 11 trees, at a cost of $190, only 40% of the cost of measuring all the trees, a 1:2.5 cost ratio.

If one had measured all of the 184 trees for VBAR at OSU, the resulting combined SE% would have been 7.5%, with a total cost of $488 (Table 3). Measuring only about half of the trees on either the even- or odd-numbered plots would have only slightly increased the combined SE%, to 7.6%, but reduced the total cost to between 59% and 66% of the cost of measuring all the trees. By using a 57.59 m^2/ha BAF to select 39 (21%) VBAR measurement trees, the combined SE% increased a little, to 7.8%, but at 41% of the cost of the standard method. The same 7.5% combined SE% could also have been obtained by using about 29 count plots and measuring about 15 trees, at a cost of $204, which is 42% of the cost of the standard method, a 1:2.4 cost ratio.

We can demonstrate these calculations using the OSU example, data from measuring all trees, and eqs. 3, 4, and 5:

$$\begin{aligned} P_{\mathrm{TC/VBAR}} &= \frac{33.0\% \times \sqrt{\$2 \text{ per VBAR}}}{16.1\% \times \sqrt{\$2 \text{ per TC}}} \\ &= 2.05 \text{ basal area plots per VBAR tree} \end{aligned}$$

$$\begin{aligned} n_{\mathrm{VBAR}} &= \frac{33.0\%^2 + (16.1\%^2 \times 2.05_{\mathrm{TC/VBAR}})}{7.8\%^2 \times 2.05_{\mathrm{TC/VBAR}}} \\ &= 14.05 \text{ VBAR trees} \end{aligned}$$

$$\begin{aligned} n_{\mathrm{TC}} &= 2.05_{\mathrm{TC/VBAR}} \times 14.05 \text{ trees} \\ &= 28.81 \text{ basal area plots} \end{aligned}$$

In application, these numbers would likely be rounded up. This would suggest a sampling design that measured tree counts for basal area on at least 15 plots and measured at least 29 trees for VBAR, as reported in the results above. For the Tahoe data, the same numbers would be 2.67 basal-area plots per VBAR trees with 27.0 count plots and 10.1 trees measured for VBAR.

An alternative way to compute the optimum sample would be to consider the costs of traveling between plots and of measuring basal area (tree counts) as a variable cost of $6.00 per plot. The cost to measure selected trees would still be $2.00 per tree. Although the general results would be similar to those given above, this assumption would cause an increase in the cost of measuring basal-area (tree-count) plots and would shift the optimal solution to one of measuring fewer basal-area plots and more VBAR trees. For the Tahoe data, this would result in 23.9 tree-count plots and 15.5 VBAR trees and total cost of $270 (1:1.8 cost ratio, i.e., comparing cost with that of measuring all trees). At OSU, the results would be 24.8 tree-count plots and 21.0 VBAR trees and total cost of $191 (1:2.6 cost ratio).

For those who might be concerned with dropping the third term when calculating the combined SE% with eq. 2, we report that the result would have been a reduction in SE% by

0.084% and 0.025% for the Tahoe and OSU cruises, respectively, using the method of Goodman (1960). Given the practice of rounding up the sample size (or even padding it a bit for insurance), the truncated form of eq. 2 is quite adequate for planning and reporting.

Discussion and application

For both of these distinctly different timber types, basal area (tree count) was more variable (higher CV) than VBAR, showing that more effort should be put into taking tree counts, rather than into measuring trees. This would almost always be the case, even in plantations. The optimum ratio of tree-count plots to VBAR-measured trees was about 2.7 at Tahoe and 2.1 at OSU (eq. 3). In traditional applications that measure all the trees on a plot, this would entail measuring all the trees on about two or three plots, assuming the same BAF angle gauges are used for selecting trees. Most people would feel uncomfortable with this level of sampling because so few locations are sampled and they are so far apart. The more likely practice would be to measure all the trees on every third or fourth plot, therefore oversampling VBAR and increasing the total cost, when only about one tree needs to be measured for VBAR on every two or three plots.

The big BAF method of selecting trees for VBAR measurement is applied at a sample point by first taking a sweep with a "normal" sized BAF to get a tree count to calculate stand basal area. Next, a second sweep is taken at the same sample point with a larger BAF to determine which trees to measure for VBAR and tree-value information (actually the two sweeps could be done at different locations, but this is not necessary). For example, if a 6 m^2/ha BAF is used to get tree count and the goal is to measure about 1/10 of the sample trees for VBAR, a 60 m^2/ha BAF angle gauge could be used for the second sweep.

If the average tree count of about 9 trees per plot is reasonable at Tahoe and OSU, the above cost assumptions would suggest obtaining VBAR measurements on about 11 of the expected 252 trees on 28 plots at Tahoe and about 15 of the expected 261 trees on 29 plots at OSU. This would require using about a 105.2 m^2/ha BAF (252/11 trees × 4.592 BAF) at Tahoe and about a 159.8 m^2/ha BAF (261/15 trees × 9.184 BAF) at OSU.

How is it possible to get large BAFs to use for this procedure? Large BAFs are easy to get with relascopes. The largest BAF on the American scale is 82.6 m^2/ha and is obtained by using the bars between the "0" on the left side of the scale and the "d" on the right side of the scale. Other large BAFs for this scale are 20.66 m^2/ha ("0–b") and 57.39 m^2/ha ("0–c"). The wide-scale relascope offers the widest range of BAFs, where the BAF, in square metres per hectare, is equal to the number of bars squared. The largest BAF on the wide-scale relascope is 144 m^2/ha and is obtained by using all 11 of the full bars and the 4 quarter bars. Finally, the metric CP (correction percentages) scale relascope has a largest BAF of 115.17 m^2/ha, using nine large bars and the four small bars on the right side of the scale. Prisms are also commonly used as angle gauges and can sometimes be specially ordered as 20.66 m^2/ha or 22.96 m^2/ha BAFs. To get even larger BAFs using prisms, one can tape two prisms together; the resulting BAF can be computed by

$$\mathrm{BAF}_{\mathrm{combined}} = (\sqrt{\mathrm{BAF}_1} + \sqrt{\mathrm{BAF}_2})^2$$

For example, combining two 10 m^2/ha BAF prisms would give a BAF of 6.32 m^2/ha. The small "error" that arises from taping two prisms together is of no consequence, as the device is just being used to select trees. If one wanted the correct BAF, the homemade prism could be calibrated (see Bell and Dilworth 2002).

A homemade stick-type angle gauge can also be easily made for any BAF. This requires knowing the desired BAF and the distance the "target" will be held from the eye. If the distance (D) is in centimetres and the BAF is in square metres per hectare, then the width (W) of the target, in centimetres, would be as follows:

$$W = \frac{D \times 2}{\sqrt{(10\,000/\mathrm{BAF}) - 1}}$$

If the BAF is in square feet per acre and D is in inches, replace 10 000 above with 43 560 to get the width of the target in inches. For example, to get a 115 m^2/ha (502 ft^2/acre) BAF, one could attach a 12.94-cm-wide target to the end of a 60.0-cm-long stick or chain, which could be held up to the eye. If no angle gauge is available, the limiting distance to trees could be measured and checked using the plot-radius factor (PRF) for the BAF being used. The PRF is the ratio of the borderline distance to the tree, given its diameter (see Bell and Dilworth 2002), and can be computed as the distance to the center of the subject tree in metres per centimetre of tree diameter, given the BAF in square metres per hectare, using the following formula:

$$\mathrm{PRF}_{\mathrm{c}}\ (\text{metres}) = \frac{0.05}{\sqrt{\mathrm{BAF}}}$$

If the BAF is in square feet per acre, then the 0.50 above would be replaced by 8.696 26 to compute the PRF as feet of distance per inch of tree diameter. It is sometimes more convenient to measure limiting distance to the face of a subject tree, rather than trying to estimate where the center is. In this case, the PRF to the tree face can be computed in metres by using the following (for a discussion of stem out-of-roundness issues associated with this, see Iles and Fall 1988):

$$\mathrm{PRF}_{\mathrm{f}}\ (\text{metres}) = \mathrm{PRF}_{\mathrm{c}}\ (\text{metres}) - \left(\frac{1}{200}\right)$$

For PRF in feet, replace the 200 above with 24.

The big BAF method has several advantages. First and foremost, selecting measured trees with the big BAF method is simple, and it is easier to check cruise than selecting trees at random or systematically. Second, it distributes the measured trees throughout the area, which is more statistically efficient than measuring them in clusters, and this makes users much more comfortable. Third, it allows cruisers to better optimize their samples by recognizing the different roles of estimating basal area by tree counts and estimating VBAR from tree measurements based on their respective variation. This can lead to large cost savings. The large BAF selects trees less often, but each diameter class is selected in the same proportion. The big BAF method can also provide a separate estimate of basal area. The basal-area estimates for the two BAFs used should be approximately the same over a number of cruises. If it is not, it might suggest that crews are missing trees, which might mean the need to consider the use of a larger counting BAF to reduce the apparent tree-count errors caused by brush, distance, or edge effects or the need for more training. Finally, the big BAF method does not require any special computational techniques.

Conclusion

Separating the process of variable plot sampling into its components — measuring basal area by counting trees and measuring trees to get VBAR information — provides the advantage of balancing the sampling effort given to each. Measuring all of the trees on every third or fourth plot is an improvement over measuring all “in” trees. The two case-study examples demonstrate that a second, larger BAF angle gauge can be used to quickly select trees to be measured, and this is an easy alternative that provides for a better geographic distribution of trees, statistically efficient sampling, and easy auditing.

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